

M.Sc.(AI)/02.22.001 Reg. No.

--	--	--	--	--	--	--	--

A

COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY

**M.Sc. COMPUTER SCIENCE WITH SPECIALISATION IN ARTIFICIAL INTELLIGENCE
FIRST SEMESTER EXAMINATION, FEBRUARY 2022**

21-344-0101 Mathematics for AI

Time: 3 Hrs.

Maximum Marks: 50

Answer any five questions

Each section carries 10 Marks

QUESTIONS		MARKS
1.	<p>i. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$ and $D = \{7, 8, 9\}$. If the universe is $U = \{1, 2, \dots, 10\}$, find:</p> <p>a) $A \cap D$ b) $(D \cap C^c) \cup (A \cap B)^c$ c) $B \oplus C$ d) $\emptyset \cup C$</p>	2
	<p>ii. What is a partially ordered relation? Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$. Consider a partial order relation, "x divides y". Draw the Hasse diagram of the poset (A, \leq).</p>	3
	<p>iii. Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) / x - y \text{ is divisible by } 3\}$. Is R an equivalence relation or not. Draw the graph of R.</p>	5
2.	<p>Let $u_1 = [3 \ -1 \ 2]^T$ and $u_2 = [3 \ 1 \ 5]^T$.</p> <p>i. Express the vector $v = [9 \ 11 \ 27]^T$ as a linear combination of u_1 and u_2 if possible.?</p>	5
	<p>ii. Let $u_1 = [0 \ -5 \ 5]^T$, $u_2 = [0 \ 3 \ -3]^T$, $u_3 = [1 \ 1 \ 1]^T$, $u_4 = [1 \ 0 \ 1]^T$, $u_5 = [2 \ 2 \ 0]^T$. Determine whether these vectors are linearly independent or not.</p>	5
3.	<p>i. Find the eigenvalues and eigenvectors of the matrix?</p> $A = \begin{pmatrix} 7 & 0 & 3 \\ -3 & 2 & -3 \\ -3 & 0 & -1 \end{pmatrix}.$	5

	ii. Find the rank of the matrix and determine the type of solution? $A = \begin{pmatrix} 7 & 0 & 3 \\ -3 & 2 & -3 \\ -3 & 0 & -1 \end{pmatrix}.$	5
4.	Consider the Following functions $f_1(x) = \sin x_1 \cdot \cos x_2 \quad x \in \mathbb{R}^2 \quad f_2(x) = x^T y \quad x, y \in \mathbb{R}^n$ i. What are the dimensions of $\partial f_i / \partial x$?	2
	ii. Compute the Jacobian of $f_1(x)$.	3
	iii. Compute the gradient of w $w = ux^2 + uy^2 + uz^2$ $x = \delta \sin \varphi \cos \Theta$ $y = \delta \sin \varphi \sin \Theta$ $z = \delta \cos \varphi$	5
5.	i. Calculate the Coefficient x^5 in $(3x^2 - 2/x)^{15}$.	3
	ii. Solve the following recurrence relation using generating functions. $a_{n+1} = a_n = 3^n, \quad n \geq 0 \quad \text{where} \quad a_0 = 1$	5
	iii. Find out the recurrence relation for $a_{n+1} = 4a_n$, where $n \geq 0$ and assume that $a_0 = 3$.	2
6.	i. Compute the distance between two vectors: $X = [1 \ 0 \ 5]^T, \quad Y = [0 \ 2 \ 4]^T$	2
	ii. Let $u = [7 \ 1]^T$ and $v = [5 \ 5]^T$. Find the angle between u and v .	2
	iii. (a) Define orthonormal vectors. (b) Find the projection $\pi_u(x)$ of the vector x onto the plane of b_1 and b_2 , when $x = [1 \ 0 \ 0 \ 1]^T$ $b_1 = [1 \ 7 \ 1 \ 7]^T \quad \& \quad b_2 = [-1 \ 7 \ 1 \ -7]^T$ (c) What projection matrix P_π will produce the projection $\pi_u(x)$ $= P * x$ for every vector x in \mathbb{R}^4 ?	6
7.	i. Define Quantifier. What are the different types of quantifiers? Give an example for each.	2.5

	ii. Prove that following statement is a tautology: $(r \rightarrow s \wedge t) \rightarrow (\neg t \rightarrow \neg s)$	2.5
	iii. Check the validity of the following argument: "If I go to college then I attend all classes. If I attend all classes, then I get S grade. I do not get S grade and I do not feel happy." Therefore, "If I do not go to college then I do not feel happy".	5